Tactical Missile Warheads, J. Carleone, ed.
Prog. in Astronautics and Aeronautics, Vol. 155
AIAA, Washington, 1993

LA-UR-87-1670

Chapter 12
Part B

Special Topics in Test Methodology: Tomographic Reconstruction of Axially Symmetric Objects from a Single Dynamic Radiograph

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1. Introduction

IT is possible to reconstruct the cross section of a three-dimensional, axially symmetric object from a single radiograph, as depicted in Fig. 1. This method of analysis has been applied to many problems in many different fields in the past, including several recent applications to static and dynamic testing at Los Alamos.[1–3] In industrial radiography, where many objects have nearly circular symmetry, such an approach offers significant benefits as an image analysis tool. These benefits include improved delineation of material boundaries, enhanced display of deviations from axial symmetry (defects, for example), and estimation of the radial dependence of the attenuation coefficients of the materials. It is often possible to observe in the reconstruction subtle features of the object that are unobservable in the original radiograph. The improvements brought about by the tomographic method are due to the effective removal of overlying material from the radiograph, which allows the reconstruction to be displayed with an increase in contrast.

2. Method

First, consider the reconstruction from its projection of the radial dependence of a two-dimensional object that possesses circular symmetry. The projection of a function $f(x, y)$ of the two spatial variables x and y is defined as

$$p(R) = \int_L f(x, y)\, \mathrm{d}s \tag{1}$$

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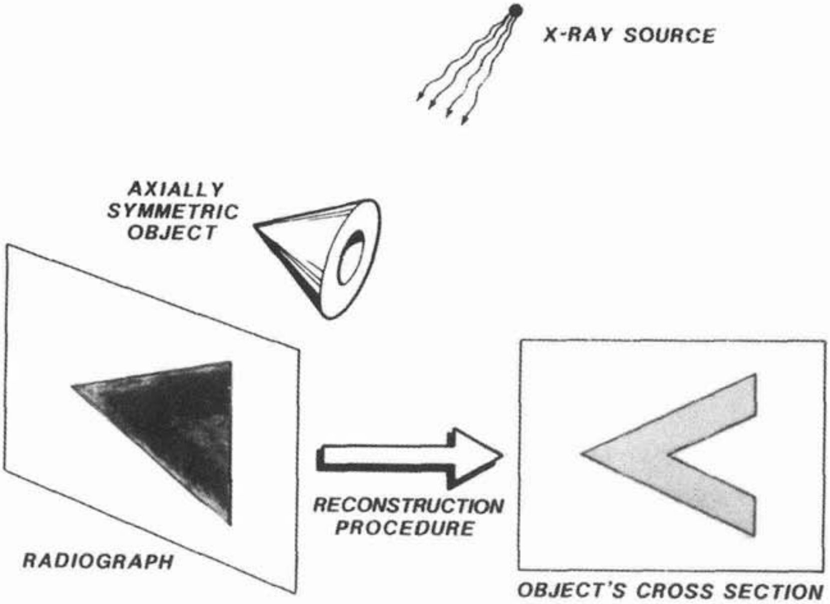

Fig. 1 Overview of the tomographic method. An object with axial symmetry is radiographed with the radiographic axis perpendicular to the axis of symmetry. The reconstruction procedure transforms the radiograph into an estimate of the cross section of the object.

where the line integral is taken along the straight line L, which is specified by its distance to the origin R. This projection is given only as a function of R because $f(x, y)$ is assumed to have circular symmetry about the origin. The solution for f given a known projection $p(R)$ was provided by Abel in 1826[4] and has been elaborated upon even in recent years.[5–8] An alternative approach avoids the mathematical difficulties that arise when one tries to evaluate the Abel inversion. In this approach, the density function $f(x, y)$ of each cross section of an object with axial symmetry is considered to be composed of a series of nested annuli, as depicted in Fig. 2. The outermost annulus, the projection of which is shown in Fig. 2, is the only annulus that contributes to the ends of the projection interval. Thus, it is possible to determine the amplitude of the outermost annulus from these projection values and consequently its contributions to each inner projection sample. The same analysis is applied to the next annulus, and so on until the center is reached. This procedure is analogous to peeling an onion by removing one layer at a time.

A minor variation on this model of nested annuli is useful in situations in which the data on the right side of the projection differ from the data on the left side. Each annulus is replaced by two annuli with tapered density profiles. The density of one of these annuli varies linearly from zero on the right side to full density on the left side; the density of the other varies linearly in the

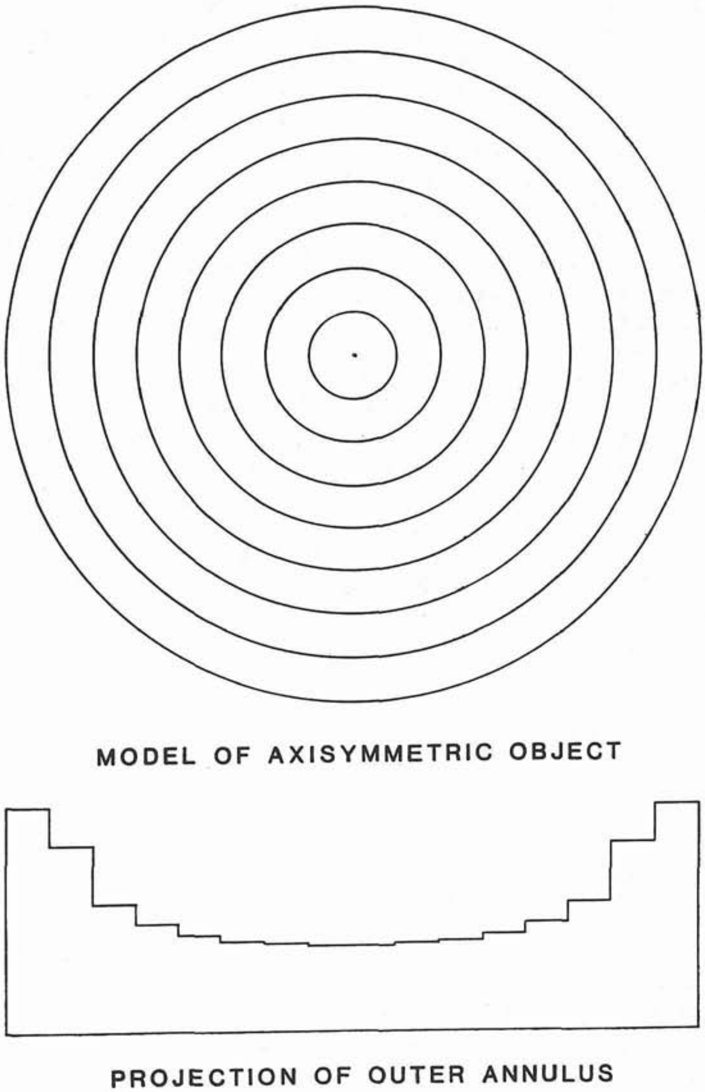

Fig. 2 Model of a two-dimensional object with circular symmetry. The model is composed of a series of annuli whose widths correspond to the width of a pixel in the digitized radiograph. Below is the projection of the outer annulus.

opposite direction. The amplitudes of the first are determined by the projection data on the left side. The amplitudes of the second are determined by the data on the right. Then the major contribution to the reconstruction on each side of the rotation axis comes predominately from the data of the corresponding side of the projection. This model of annuli with tapered densities successfully avoids discontinuities at the center that might otherwise result from inconsistent data.

Nearly 70 years ago, Radon[9] showed that it is possible to determine completely an arbitrary two-dimensional function from a set of line integrals, provided they are known for all lines that cross the region in which the function is nonzero. In practice, the tomographic reconstruction of an arbitrary two-dimensional object normally requires projection measurements at a large number of angles. Zoltani et al.[10,11] have proposed the use of multiple pulsed-x-ray sources and detectors to obtain many simultaneous views of a dynamic event to provide the data necessary to reconstruct objects without symmetry. Clearly the existence of axial symmetry and the consequent reduction in the number of required views to a single one is of great benefit. The possibility that a function may be reconstructed from multiple projections has been put to use in the relatively new technique called computed tomography (CT) or computerized axial tomographic (CAT) scanning. The striking success of this revolutionary technique in medical radiography is now legendary.

Suppose that a three-dimensional object is radiographed with an essentially monoenergetic x-ray source. If the x rays are detected by the direct-recording film, the optical density of the developed film is proportional to the x-ray intensity. The film's optical density at the intersection of each line L, which originates at the point source, with the film plane is given by

$$D_L = D_0 + D_1 \exp(-p_L) \tag{2}$$

where the path length

$$p_L = \int_L \mu(x, y, z) \mathrm{d}s \tag{3}$$

is the line integral along line L of the object's linear attenuation coefficient distribution $\mu(x, y, z)$, evaluated at the energy of the x rays used. In Eq. (2), D_0 is the background density of the film at that point, which includes the fog value of the film and any contribution from scattered radiation; D_1 is the net density (above D_0) that would be obtained in the absence of the object. Thus, the projections needed as input for tomographic reconstruction, Eq. (1), may be obtained from radiographic measurements. Inversion of Eq. (2) yields the path length in terms of measured film density D_L:

$$p_L = -\ell n \frac{(D_L - D_0)}{D_1} \tag{4}$$

From the previous discussion, it can be seen that tomographic reconstruction based upon projections obtained in this way will yield the linear attenuation-coefficient distribution of the object.

Now consider a three-dimensional object that possesses axial or rotational symmetry. In any plane that is perpendicular to the symmetry axis, the object's cross section has circular symmetry. If the object is radiographed with a set of parallel x-ray beams, equivalent to a point source placed infinitely far from the object, all of which lie in such a plane, the path lengths obtained from applying Eq. (4) will correspond precisely to those considered in the two-dimensional example above. Thus, the radial distribution of that cross section can be reconstructed using the same technique. A line-by-line analysis carried out in this way on each line of the radiograph that is perpendicular to the symmetry axis yields the complete radial distribution of μ for the object. This line-by-line analysis is legitimate because the contributions to each line come from one, and only one, cross-sectional plane through the object, and each cross section has circular symmetry. In practice, the x-ray source cannot be placed infinitely far from the object. Indeed, often it must be placed very close. In this case, the line-by-line analysis is not strictly valid because the contributions to each line of the radiograph, with the possible exception of a single line, come from neighboring cross sections. Owing to its simplicity, the line-by-line method is employed in the present analysis even though it may not be exactly correct. The consequences of this approximation are not severe, as shown in Ref. 3.

3. Results

A test object with axial symmetry was constructed to illustrate the usefulness of the tomographic analysis technique. The object is a 70-mm-long right circular steel cylinder, 120 mm in diameter, with a 45-deg cone removed from one end to a depth of 40 mm. Four 2-mm-square grooves were machined on the flat face as well as on the inside of the cone at radii of 10, 20, 30, and 40 mm. A 2-mm-diam, 2-mm-deep hole was also drilled into the center of the flat face. This object was radiographed with a 120-Ci Co^{60} source for 15 min using Kodak AA film placed in close contact with 0.25-mm-thick lead screens, front and back. The axis of symmetry of the cylinder was perpendicular to the radiographic axis. The source diameter was about 1.5 mm. In order to closely approximate the parallel beam geometry that is assumed by the reconstruction procedure, the source was placed 4.5 m from the object, which was in turn 0.5 m from the film. The film was scanned on a Perkin-Elmer/PDS-2020 microdensitometer to produce a digital image that was 425 by 275 pixels in size with a pixel spacing of 0.4 mm. Figure 3a displays the resulting digitized radiograph. The diffuse densities in this radiograph range from about 0.35 to 1.8, the latter corresponding to the total density produced by the unattenuated x-ray beam.

When the tomographic analysis outlined above is performed on each line of the radiograph, Fig. 3a, and the reconstructed radial profiles are combined to form an image, Fig. 3b results. The entire computation time to obtain this result was 9 min of CPU time on a DEC VAX-11/785. This reconstruction closely resembles the cross section of the object itself. The value of the reconstruction in the region of the steel is very nearly the same as the tabulated value[12] of the linear attenuation coefficient for iron at 1.25 MeV, namely 0.042 mm^{-1}. The grooves, which were very difficult to observe in the original radiograph, are now easily seen. In the original radiograph, the grooves on the flat face

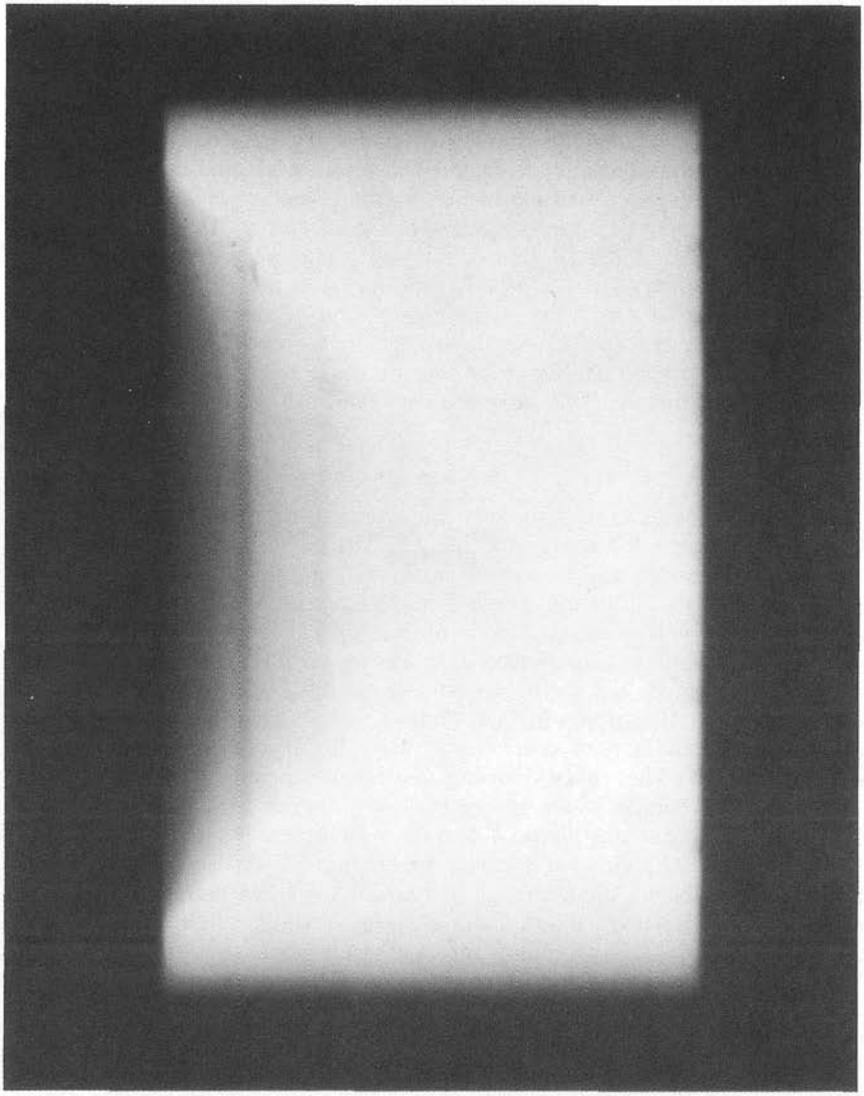

Fig. 3a Radiograph of an axially symmetric steel object taken with a Co^{60} source using AA film placed between two 0.25-mm-thick lead screens.

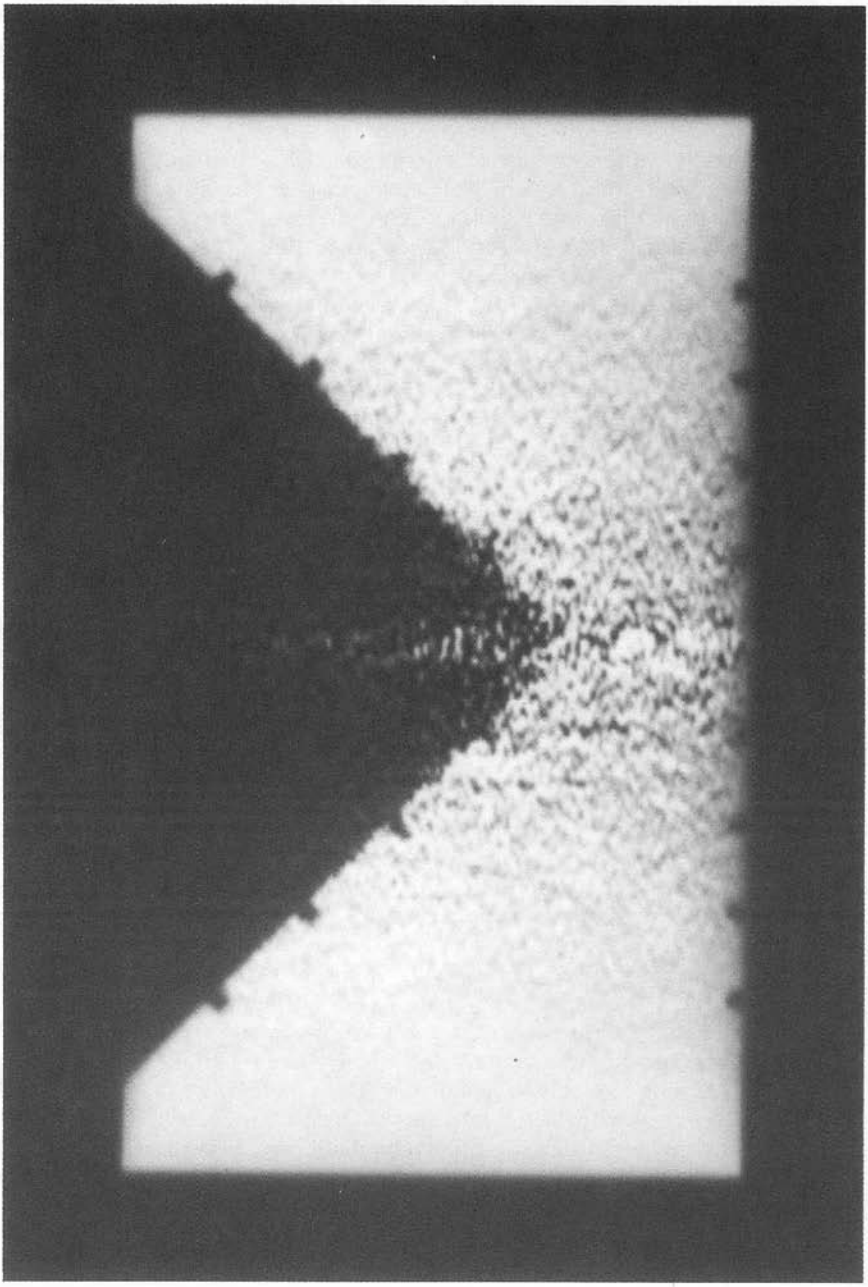

Fig. 3b Reconstruction of the cross section of the object derived from Fig. 3a. The 2- × 2-mm grooves machined in the inner cone and on the end face are much more readily visible. These grooves were nearly invisible in the original radiograph.

of the cylinder produced fluctuations in diffuse optical density ranging from 0.04 for the 10-mm-radius groove to 0.11 for the 40-mm one. These correspond to a range in contrast of luminance from 10% to 27%. Although a 10% contrast is readily observed under the best of viewing conditions,[13] it is essentially impossible to observe in the original radiograph, where the edge of the cylinder creates a rapidly falling luminance profile. On the other hand, this groove is easily seen in the reconstruction, which demonstrates that it is the noise in the original radiograph that ultimately limits one's ability to derive information from it. This is one of the major goals of image processing.

The severe enhancement of the noise near the axis of symmetry arises from a nearly singular condition in the reconstruction procedure there. This effect is due to the fact that as a ring of material with constant thickness decreases in diameter, it produces a smaller and smaller optical-density signal on the radiograph, making it increasingly difficult to observe in the presence of nearly constant film-density noise. The uncontrolled growth of noise in the reconstruction poses a problem in interpretation of the image. One method to control the noise in the reconstruction is nonlinear maximum a posteriori probability (MAP) restoration, introduced to image processing by Hunt.[14] The application of this iterative approach to the problem of reconstruction of axially symmetric objects has been reported elsewhere.[15,16]

Figures 4a and 4b show the result when the tomographic procedure is applied to the radiograph of an experimental, explosively formed copper penetrator. The void in the center of the penetrator is delineated much better in the reconstruction than in the radiograph. In dynamic radiography such as this, the reconstruction method provides a means to estimate the densities of the object in regions of mixed materials or to observe slight density fluctuations due to the presence of shock waves. The application of tomographic analysis to the radiographs of other test objects has shown that it greatly enhances the visualization of minor defects in the objects and that defects not possessing axial symmetry remain localized in the reconstruction.

Two technical points are worth making. The highly nonlinear behavior of Eq. (4) near $D = D_0$ makes the reconstruction there very sensitive to the choice of D_0. Therefore, D_0 must be known very accurately if serious systematic errors in the reconstruction are to be avoided. Slight variations in D_0 arising from variations in the scattered radiation field may be difficult to take into account.[3] Another difficulty in tomographic analysis is the assumption of a monochromatic source. Polychromatic x-ray sources are widely used. The attenuation law for these is not as simple as given in Eq. (2). The problem is exacerbated by the possible presence of many different kinds of material in industrial radiographic investigations. Whereas the slight polychromatic and beam-hardening effects present in medical CT scanners have been successfully eliminated,[17] the effects in industrial radiography can be much more significant and may be harder to cope with.

4. Conclusions

It has been shown that the extension of the Abel inversion, the reconstruction of a three-dimensional axially symmetric object from a single radiograph taken

Fig. 4a Radiograph of an explosively formed copper penetrator. The penetrator, supplied by Honeywell Corp., was radiographed with a 50-ns-long pulse produced by a 0.8-MeV electron beam. Its velocity was 2.8 mm/μs.

Fig. 4b Reconstruction of the cross section of the object derived from Fig. 4a. It shows the void in the middle of the penetrator much more clearly. Hence, the solution is easily realized by peeling the onion from the outside to the inside.

with the radiographic axis perpendicular to the symmetry axis, offers significant benefits as an image analysis tool. These benefits include improved delineation of material boundaries; enhanced display of minor deviations from axial symmetry, as produced by defects; and accurate estimation of the linear attenuation coefficients of the materials. Frequently, this technique allows observation of features in the object that are too subtle to be seen in the original radiograph.

Acknowledgments

The author would like to express his appreciation to George Brooks for providing the radiographs used in the static example (Fig. 3a). Rollin Whitman performed the reconstruction of this object. The reconstruction of the dynamic radiograph (Fig. 4b) was supplied by Karl Mueller, Larry Schwalbe, and Michael Christian. This work was suppoorted by the U.S. Department of Energy under contract W-7405-ENG-36.

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