Tomographic reconstruction using 3D deformable models

X L Battle‡, K M Hanson§ and G S Cunningham||

DX3, Los Alamos National Laboratory, Los Alamos, NM, 87545, USA

Abstract.

We address the issue of reconstructing the shape of an object with constant interior activity from a set of projections. We directly estimate the position of a triangulated surface describing the boundary of the object by incorporating prior knowledge about the unknown shape. The inverse problem is addressed in a Bayesian framework. We compute the reconstruction with a maximum *a posteriori* (MAP) estimate. The Adjoint differentiation technique computes the derivatives needed for the optimization of the model parameters. We present results from a numerical simulation study of dynamic cardiac imaging. We simulate a first-pass exam using the measured system response of the University of Arizona FASTSPECT imager and a numerical phantom of the right ventricle. We reconstruct the shape of the ventricle from several views of emitted gamma rays with very few counts. The comparison of our method with activity-based estimation methods shows the usefulness of the deformable model approach.

Short title:

July 1, 1999

‡ Present address : Laboratoire de Biophysique, Université de Bretagne Occidentale, BP 815, 29285 Brest Cedex

§ kmh@lanl.gov

|| cunning@lanl.gov

1. Introduction

Deformable models are commonly used in medical imaging for tasks such as segmentation and restoration (Staib *et al* 1996). These models can also be efficiently used to solve ill-posed tomographic reconstruction problems. Because the geometrical structure of an object is often known a priori, Bayesian methods can substantially improve the accuracy of the reconstructions obtained from limited data. Hanson *et al* (1996c, 1996d, 1994, 1993a and 1993b) and Cunningham *et al* (1994a and 1994b) have developed a framework in which such analysis can easily be conducted. The Bayes Inference Engine (BIE) is a flexible software tool that allows one to interactively define models of radiographic measurement systems and geometric models of objects. Based on an object-oriented approach, the BIE allows one to design a measurement, define a geometric parameterization for the models and optimize a likelihood or posterior with respect to the user-selected parameterization. This optimization problem would be intractable without the knowledge of the gradient of the posterior. Adjoint differentiation (Hanson *et al* 1993a) addresses this issue in a very efficient and elegant way. In this paper, we illustrate these methods by addressing the issue of reconstructing the external shape of an object of constant interior activity from a set of projections. We recall the principles of Bayesian analysis and present the specific characteristics of this 3D problem as well as the principles of adjoint differentiation. We discuss the choice of an appropriate model and a meaningful prior, as well as the correct likelihood function. Based on the results of a numerical simulation, we show the value of these methods in specific contexts such as SPECT dynamic cardiac imaging. The results of the reconstructions from first-pass data show the advantages of the method in high-noise situations compared to reconstructions from classical activity-based methods.

2. Bayesian approach

The Bayesian framework is a model-based framework in which a priori knowledge is integrated. Given some data d, the probability of an hypothesis x is given by Bayes' law :

$$P(x \mid d) = P(d \mid x) \times P(x) \tag{1}$$

$P(x \mid d)$ denotes the posterior probability, $P(d \mid x)$ denotes the likelihood of the hypothesis x to the data d, and $P(x)$ denotes the prior probability of x. The maximum *a posteriori* (MAP) estimate is the value of x that maximizes $P(x \mid d)$.

$$map = argmax[P(x \mid d)] \tag{2}$$

In the problem addressed here, we try to reconstruct the shape of an unknown object from a set of projections (the real data). The unknown shape is represented by a geometrical model and constitutes our hypothesis. The simulation of the measurement process produces the predicted data. The comparison of the predicted data with the real data gives the likelihood of our hypothesis. Prior knowledge is incorporated as another function of the hypothesis to produce the posterior function. An optimization allows us to determine the MAP estimate. In practice, we deform the shape to best match the real data. The following sections discuss these aspects in more detail.

2.1. Model

We focus on objects of constant interior amplitude for which the shape is of interest. We therefore represent the shape of the external surface rather than the full volume. There are numerous ways to represent a 2D surface in 3D space, from simple triangulations to complex NURBS surfaces. We choose to represent the objects by a closed, finely-divided triangulated surface enclosing a region of constant interior amplitude. The simplicity of the structure has several advantages. The triangulated surface is the most flexible model to represent a surface in space. By increasing the resolution (number of vertices and size of the triangles) one can represent any surface of any kind, even the most contorted. As a consequence, no smoothness constraint is embedded in the model itself whereas spline surfaces, for example, embed a fairly strong smoothness constraint. From a Bayesian point of view, a model free of constraints is a crucial consideration since it allows total freedom in the definition of the prior (Hanson *et al* 1997 and 1996a). Moreover, any other representation can be reduced to a triangulated surface. Therefore, this choice does not limit the scope of the approach. The triangulated surface is not only the most flexible, but also the simplest. As a consequence, the algorithms developed for this structure remain fairly simple, which is an important consideration when adjoint differentiation is used.

2.2. Likelihood

In nuclear medicine images the number of gamma rays detected in each pixel follow a Poisson probability distribution. The Poisson likelihood function is not defined for nonzero measured counts when the expected number is zero. To overcome this ambiguity, we approximate the Poisson likelihood with a weighted chi-squared likelihood wih the variance set to $n + 1$, where n is the number of counts in the considered pixel.

2.3. Prior

The intended application is the reconstruction of the shape of the blood pool within the heart. As expected in dynamic SPECT, the amount of noise and the small number of counts will significantly alter the quality of the projection data. Without a regularization term, the reconstructions ought to be jagged and spiky. We therefore incorporate some prior knowledge about the expected degree of smoothness of the shape to reconstruct. It is indeed reasonable to assume that the walls of the heart are fairly smooth. A good measure of the smoothness of a continuous curve is given by the integral of the curvature squared along the curve (Hanson *et al* 1997, 1996a and 1996b). This notion can be adapted to the case of a segmented curve by locally fitting a quadratic curve to two consecutive segments. The generalization of this result to surfaces is not simple because of the tensorial nature of the curvature. As a first approximation, we compute a measure of the smoothness of the triangulated surface by comparing the directions of normals of neighbouring triangles. The expression we retained is the following.

$$(\textstyle\sum_{i\in T}\sum_{j\in N(i)} \tan^2(\frac{\theta_{i,j}}{2}) \times S_i) \times (\sum_{i\in T} S_i)^{-1} \qquad (3)$$

T denotes the set of the triangles, $N(i)$ denotes the set of the neighbours of the triangle of index i, $\theta_{i,j}$ denotes the angle between the normals of triangles of indices i and j and S_i denotes the area of the triangle of index i.

3. Adjoint differentiation

3.1. Need

The MAP estimate is the solution that maximizes the posterior probability, as shown in equations 1 and 2. Maximizing the posterior is equivalent to minimizing the minus log posterior function :

$$-\ln P(x \mid d) \sim -\ln P(d \mid x) - \ln P(x) \qquad (4)$$

or

$$\psi = \varphi + \pi \qquad (5)$$

In order to balance the two terms (likelihood and prior), a factor is introduced as α in equation 5

$$\psi = \varphi + \alpha \times \pi \qquad (6)$$

α is called the strength of the prior and has a considerable influence on the result of the reconstruction. In practice, the MAP estimate is obtained by minimizing the function ψ. The minimization of this function is based on gradient-based methods. The derivatives of ψ with respect to the model parameters must be computed. The models considered require from thousands to millions of parameters, the optimization methods require from tens to hundreds of iterations. An efficient and fast way to compute the derivatives of ψ with respect to the model parameters is required. Adjoint differentiation addresses this issue in a very efficient way (Hanson *et al* 1993a).

3.2. principle

Given a sequence of transformations,

$$\mathbf{X} \overset{A}{\longmapsto} \mathbf{Y} \overset{B}{\longmapsto} \mathbf{Z} \overset{C}{\longmapsto} \varphi \tag{7}$$

where $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ are vectors and ψ a scalar, one wants to compute the derivatives of ψ with respect to the x_i in a computationally efficient way. The chain rule for differentiation gives the derivatives of ψ with respect to x_i :

$$\frac{\partial\psi}{\partial x_i} = \sum_{jk} \frac{\partial\psi}{\partial z_k}\frac{\partial z_k}{\partial y_j}\frac{\partial y_j}{\partial x_i} \tag{8}$$

The essence of adjoint differentiation is to perform the sum on the indices in equation 8 in the reverse order from that used in the forward calculation. If the sum on k is done before that on j, the sequence of calculations is the adjoint of equation 7 :

$$\mathbf{I} \overset{C'}{\longmapsto} \frac{\partial\psi}{\partial\mathbf{z}} \overset{B'}{\longmapsto} \frac{\partial\psi}{\partial\mathbf{y}} \overset{A'}{\longmapsto} \frac{\partial\psi}{\partial\mathbf{x}} \tag{9}$$

where $\mathbf{I}$ is the identity vector and $\partial\psi/\partial\mathbf{y}$ denotes the vector of term $\partial\psi/\partial y_i$. We see that adjoint differentiation computes the derivatives of ψ with respect to the model parameters in a way comparable to the forward calculation. The intermediate data-structures in equation 9 are vectors, similar to the ones in equation **??**. For each forward transformation an adjoint counterpart exists.

4. Results

In order to demonstrate the efficiency of the approach, we reconstruct the shape of a right ventricle phantom from numerically simulated data. In the context of a first-pass exam, the blood pool in the right ventricle can be considered as homogeneous and can be represented as an object with constant interior activity. We simulate an exam on the University of Arizona FASTSPECT machine. This scanner is composed of 24 gamma cameras positionned around a hemispherical array of pinholes. Each of the 24 cameras has a resolution of 64 by 64. The system response of the scanner has been measured by moving a 2mm by 2mm by 2mm (size of a voxel) source through the active volume and recording the response of each of the 24 detectors. The size of the voxel grid is 47 by 39 by 37. We use a numerical phantom of the right ventricle as our object of study. The phantom, shown on figure 1 consists in a finely triangulated surface containing 30048 triangles and 15026 vertices. We simulate an exam using 10000 counts on all the 24 detectors with a Poisson noise distribution. Such a low-count situation is expected for a single frame of an actual dynamic first-pass exam. Figure 2 shows one of the 24 projections and a profile through this projection. The number of counts per pixel varies from 0 to 9, with an average of 0.5. The reconstruction is obtained by deforming a surface to best match the data. We start with a triangulated sphere containing 2562 vertices and 5120 triangles. After about fifty iterations, the MAP estimate is obtained. Figure 1 shows several reconstructions obtained with different values of α the strength of the prior. Obviously, the choice of α has a tremendous influence on the reconstruction. In order to estimate the quality of the reconstruction, we try to represent the error on the position of the surface. Figure 3 shows contours obtained by slicing the phantom and the reconstruction in an arbitrary plane. Again, we notice the influence of the choice of α on the reconstruction. We notice that the maximum error on the position of the surface has approximately the size of a voxel (i.e. 2mm). It is difficult to compare this approach to well know iterative methods because of the great difference in the representation of the result. However, figure 3 shows a slice trough an ML-EM reconstruction of the activity after 200 iterations and the corresponding slice through the reconstruction with $\alpha = 8$. The effect of the very high noise level is striking. It is difficult to estimate where the boundary of the blood pool is in the activity-based reconstruction.

5. Conclusion

This work illustrates how deformable models can be used to solve ill-posed tomographic reconstruction problems. The principal aspects of the Bayesian approach are presented, as well as its application to 3D problems. The results show the value and the usefulness of the approach. Nevertheless, many issues need further work. The selection of the strength of the prior is a crucial issue and an automatic selection of it from the data is currently investigated with the L-curve. A prior penalizing high values of the derivative of the curvature rather than the curvature itself is investigated. The reconstruction of a real object such as the artificial heart Jarvick at the University of Arizona will allow to better validate the approach. Finally, time-evolving and variable-activity problems will also be addressed in the future.

Acknowledgments

This work was supported by the United States Department of Energy under contract W-7405-ENG-36. We are indebted to Harry Barrett and Don Wilson of the University of Arizona for supplying us with their FASTSPECT measurement matrix and a suitable simulated phantom.

References

Battle X L, Hanson K M and Cunningham G S 1997 3D Tomographic reconstruction using geometrical models *Image Processing Proc. SPIE* **3034** To be published

Cunningham G S, Hanson K M, Jennings G R and Wolf D R 1994a An object-oriented implementation of a graphical-programming system *Image Processing Proc. SPIE* **2167** 914

——1994b An interactive tool for Bayesian Inference *Review of Progress in Quantitative Nondestructive Evaluation* **14A** 747

——1994c An object-oriented optimization system *Proc. IEEE Int. Conf. Image Processing* **II** 826

Hanson K M, Cunningham G S and McKee R J 1997 Uncertainties in tomographic reconstructions based on deformable models *Image Processing Proc. SPIE* **3034** To be published

Hanson K M, Cunningham G S and McKee R J 1996a Uncertainty estimation in reconstructed deformable models *Maximum Entropy and Bayesian Methods* (M Sears) To be pubished

Hanson K M, Bilisoly R L and Cunningham G S 1996b Kinky tomographic reconstruction *Image Processing Proc. SPIE* **2710** 156

Hanson K M and Cunningham G S 1996c A computational approach to Bayesian inference *Computing Science and Statistics* **27** 202

——1996d The Bayes inference engine *Maximum Entropy and Bayesian Methods* (Hanson K M and Silver R N) 125

——1995 Exploring the reliability of Bayesian reconstructions *Image Processing Proc. SPIE* **2434** 416

Hanson K M, Cunningham G S, Jennings G R and Wolf D R 1994 Tomographic reconstruction based on flexible geometric models *Proc. IEEE Int. Conf. Image Processing* **II** 145

Hanson K M 1993a Bayesian reconstruction based on flexible prior models *J. Opt. Soc. Amer.* **10** 997

——1993b Flexible prior models in Bayesian image analysis *Maximum Entropy and Bayesian Methods* (Mohammad-Djafari A and Demoment G) 399

Staib L H and Duncan J S 1996 Model-based deformable surface finding for medical images *IEEE Trans. Med. Imag.* **15** 720

Figure 1. Phantom of the right ventricle, a triangulated surface containing 30048 triangles and 15026 vertices (a). Reconstructions obtained for $\alpha = 6$ (b), $\alpha = 8$ (c) and $\alpha = 10$ (d).

Figure 2. Projection on one of the 24 64×64 detectors (a) and profile along a column of the projection (b).

Figure 3. Slices through the original and the reconstructions for $\alpha = 6$ (a), $\alpha = 8$ (b), $\alpha = 10$ (c) and the ML-EM reconstruction (d)