Introduction to PTW

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This document summarizes the Preston-Tonks-Wallace (PTW) model, and presents some examples of typical stress-strain relations.

1. SUMMARY OF THE FORMULAS

The Preston-Tonks-Wallace (PTW) model[1] describes the plastic deformation of metals in terms of the dependence of plastic stress on plastic strain. The following summary of the PTW model is taken directly from Ref. 1.

In the PTW model, the plastic stress in a material is a function of the amount of strain ψ it has undergone, the strain rate $\dot{\psi}$, the material temperature T, and its density ρ. It is assumed that the plastic stress is independent of the history of the material, and that the plastic flow is isotropic. Material fracture or failure is not included in PTW.

The PTW model is written in terms of three scaled dimensionless variables. The scaled stress variable is

$$\hat{\tau} = \frac{\tau}{G(\rho, T)}\,, \tag{1}$$

where τ is the flow stress, which is one-half the usual van Mises equivalent deviatoric stress σ, that is, $\tau = \sigma/2$, and $G(\rho, T)$ is the shear modulus, which is a function of the material density ρ and temperature T.

The material temperature is scaled to its melt temperature T_m, which is a function of the material density ρ

$$\hat{T} = \frac{T}{T_m(\rho)}\,. \tag{2}$$

For plastic flow, clearly $\hat{T} < 1$. The equivalent plastic strain is denoted in PTW by ψ, although it is conventionally represented by ϵ. The strain rate $\dot{\psi}$ is scaled to an appropriate rate

$$\dot{\xi}(\rho, T) = \frac{1}{2}\left(\frac{4\pi\rho}{3M}\right)^{\frac{1}{3}}\left(\frac{G}{\rho}\right)^{\frac{1}{2}}\,, \tag{3}$$

where M is the atomic mass of the metal. Thus, $M = A/Avo$, where A is the atomic weight of the metal and Avo is Avogadro's constant, 6.025×10^{23} grams/mole. $\dot{\xi}$ is the reciprocal of the time for a transverse sound wave to cross an atom. The strain rate always appears in the PTW formulas in terms of the ratio $\dot{\psi}/\dot{\xi}$.

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The shear modulus is taken to be

$$G(\rho, T) = G_0(\rho)\,(1 - \alpha\,\hat{T})\,, \tag{4}$$

where $G_0(\rho)$ is the shear modulus at $T = 0$ and $\alpha > 0$ is a material parameter.

For any fixed values of strain rate and temperature, the scaled stress $\hat{\tau}$ ranges between a lower and upper limit, the yield stress $\hat{\tau}_y$ and a saturation value $\hat{\tau}_s$. The functional form for $\hat{\tau}$ depends on the strain ψ in the following way

$$\hat{\tau} = \hat{\tau}_s + \frac{1}{p}\,(s_0 - \hat{\tau}_y)\ln\left\{1 - \left[1 - \exp\left(-p\,\frac{\hat{\tau}_s - \hat{\tau}_y}{s_0 - \hat{\tau}_y}\right)\right]\exp\left[-\frac{p\,\theta\,\psi}{(s_0 - \hat{\tau}_y)\left[\exp\left(p\,\frac{\hat{\tau}_s - \hat{\tau}_y}{s_0 - \hat{\tau}_y}\right) - 1\right]}\right]\right\}, \tag{5}$$

where p and θ are material-specific parameters. The parameter s_0 is explained below.

At low strain rates, the plastic deformation process is controlled by thermal activation. The values for $\hat{\tau}_y$ and $\hat{\tau}_s$ are given by

$$\hat{\tau}_y^L = y_0 - (y_0 - y_\infty)\,\mathrm{erf}\left[\kappa\,\hat{T}\ln\left(\frac{\gamma\,\dot{\xi}}{\dot{\psi}}\right)\right], \tag{6}$$

$$\hat{\tau}_s^L = s_0 - (s_0 - s_\infty)\,\mathrm{erf}\left[\kappa\,\hat{T}\ln\left(\frac{\gamma\,\dot{\xi}}{\dot{\psi}}\right)\right], \tag{7}$$

where κ and γ are dimensionless material parameters, and the superscript L stands for low strain rates. The error function is defined as $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp\left(-t^2\right) dt$, which has the limiting values $\mathrm{erf}(0) = 0$ and $\mathrm{erf}(\infty) = 1$. Note that the logarithm is nonnegative because $\gamma\,\dot{\xi}/\dot{\psi} \geq 1$ in the low-strain-rate regime. Therefore, the argument of the erf function is nonnegative. The parameters y_0 and y_∞ are the values that $\hat{\tau}_y$ takes at zero temperature and very high temperatures, respectively; s_0 and s_∞ have analogous meanings for $\hat{\tau}_s$.

At very high strain rates, above $10^8\,\mathrm{s}^{-1}$ for example, the plastic deformation process is appropriately described by Wallace's theory of overdriven shocks in metals.[2] In this regime, the saturation stess becomes

$$\hat{\tau}_s^H = s_0\left(\frac{\dot{\psi}}{\gamma\,\dot{\xi}}\right)^{\beta}, \tag{8}$$

where the superscript H refers to high strain rate. The PTW model accounts for this transition as follows

$$\hat{\tau}_s = \max\{\hat{\tau}_s^L, \hat{\tau}_s^H\}\,. \tag{9}$$

The yield stress at very high strain rate is the same as the saturation stress, that is, $\hat{\tau}_y^H = \hat{\tau}_s^H$.

A transition is required at intermediate strain rates, so

$$\hat{\tau}_y^M = y_1\left(\frac{\dot{\psi}}{\gamma\,\dot{\xi}}\right)^{y_2}, \tag{10}$$

Table 1. PTW parameters used for copper and tantalum.[1] G_0 is given in kilobars; all other parameters are dimensionless.

Material	Cu	Ta
y_0	0.0001	0.01
y_∞	0.0001	0.00125
s_0	0.0085	0.012
s_∞	0.00055	0.00325
κ	0.11	0.6
γ	0.00001	0.00004
θ	0.025	0.02
p	2.0	0.0
y_1	0.094	0.012
y_2	0.575	0.4
β	0.25	0.023
G_0	518	722
α	0.20	0.23

where the superscript M refers to medium strain rates. The formula for the yield stress becomes

$$\hat{\tau}_y = \max\{\hat{\tau}_y^L,\, \min(\hat{\tau}_y^M, \hat{\tau}_y^H)\}\,. \tag{11}$$

2. EXAMPLES

Several examples are given in this section to elucidate the behavior of the PTW equations. The parameters used in these examples are given in Table 1, which are taken from the PTW paper.

2.1. Copper

Figures 1, 2, and 3 show the stress-strain behavior for copper in the low strain-rate regime given by Eq. (5). The flow stress has been converted to the von Mises stress using Eq. (1) and multiplying the result by 2. Parameters not mentioned in Table 1 are $T_{melt} = 1357°\mathrm{K}$, $\rho = 8.933\,\mathrm{g/cm}^2$, and atomic weight = 63.54. In addition, one needs to know that the conversion factor from bars to dynes is 10^6 (dynes/bars).

At small strains, the plastic stress in these figures rises linearly from the lower limit given by Eq. (11). As the strain increases, the stress asymptotically approaches the upper limit given by Eq. (9).

Figure 4 shows how the stress behaves over a wide range in strain rate at constant strain value. The kinks in σ_y indicate clearly the endpoints of the three strain-rate regimes. For $\hat{\tau}_y$

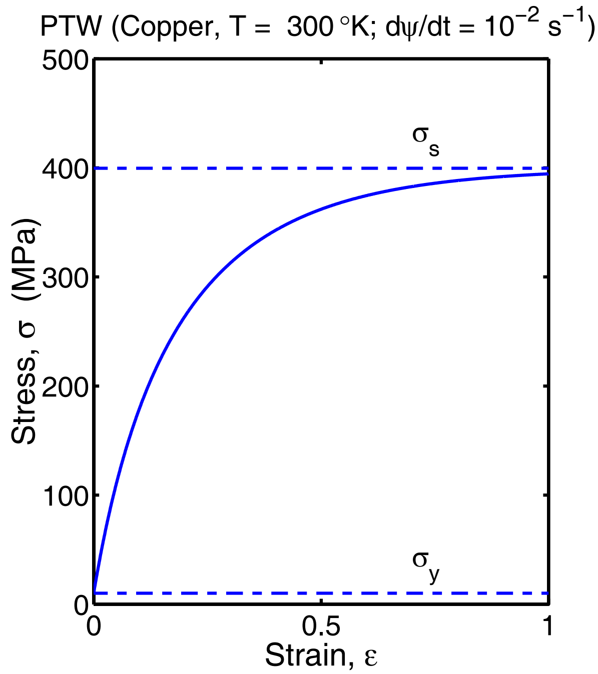

Figure 1. The von Mises stress plotted as a function of strain (denoted in the text by ψ, but often by ϵ) for copper at $T = 300\,^{\circ}K$ and $\dot{\psi} = 10^{-2}\,\mathrm{s}^{-1}$. The lower and upper limits are shown as dashed lines.

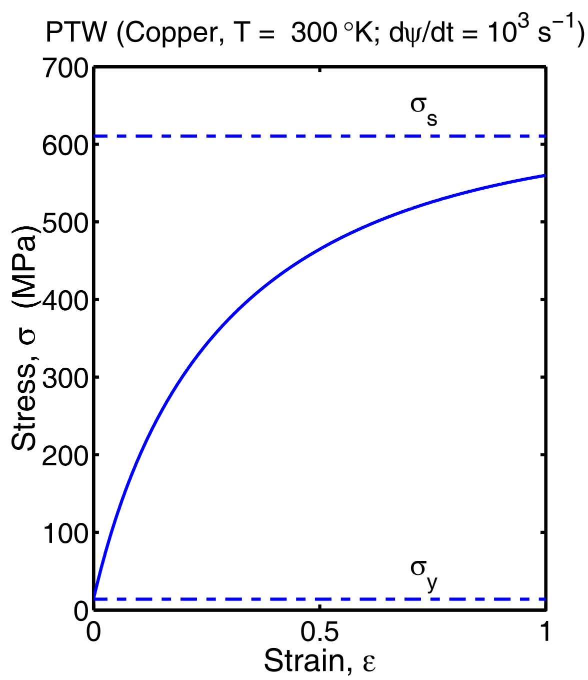

Figure 2. The von Mises stress plotted as a function of strain for copper at $T = 300\,^{\circ}K$ and $\dot{\psi} = 10^{3}\,\mathrm{s}^{-1}$. The lower and upper limits are shown as dashed lines.

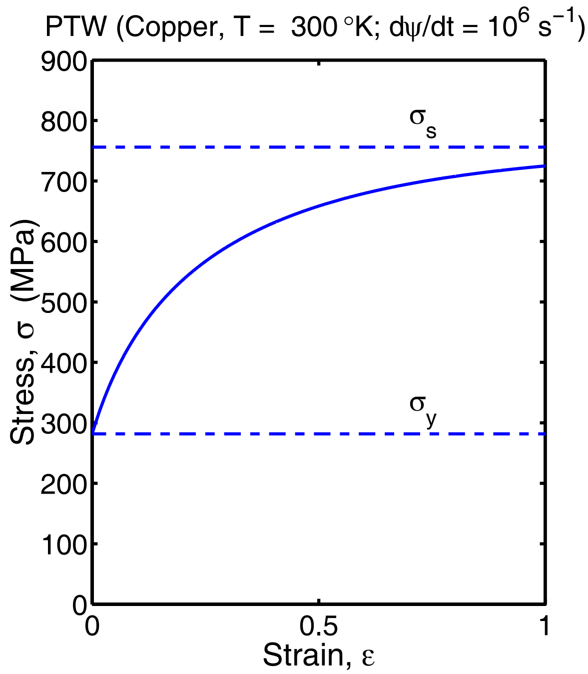

Figure 3. The von Mises stress plotted as a function of strain for copper at $T = 300\,^{\circ}K$ and $\dot{\psi} = 10^{6}\,\mathrm{s}^{-1}$. The lower and upper limits for these conditions, given by Eqs. (11) and (9), are shown as dashed lines.

the breakpoints are at strain rates of around 600 and $5{\times}10^{4}\,\mathrm{s}^{-1}$. For $\hat{\tau}_s$ the breakpoint is at the strain rate of around $10^{8}\,\mathrm{s}^{-1}$.

2.2. Tantalum

Figures 5, 6, and 7 show the stress-strain behavior for tantalum in the low strain-rate regime given by Eq. (5). The flow stress has been converted to the von Mises stress using Eq. (1) and multiplying the result by 2. At small strains, the plastic stress in these figures rises linearly from the lower limit given by Eq. (11). As the strain increases, the stress asymptotically approaches the upper limit given by Eq. (9). Parameters not mentioned in Table 1 are $T_{melt} = 3250$°K, $\rho = 16.8\,\mathrm{g/cm}^2$, and atomic weight $= 180.95$.

Figure 8 shows how the stress behaves over a wide range in strain rate at constant strain value. The kinks in σ_y indicate clearly the boundary between the low and the high strain-rate regimes. For both $\hat{\tau}_y$ and $\hat{\tau}_s$, the breakpoints occur at a strain rate of around $10^{8}\,\mathrm{s}^{-1}$.

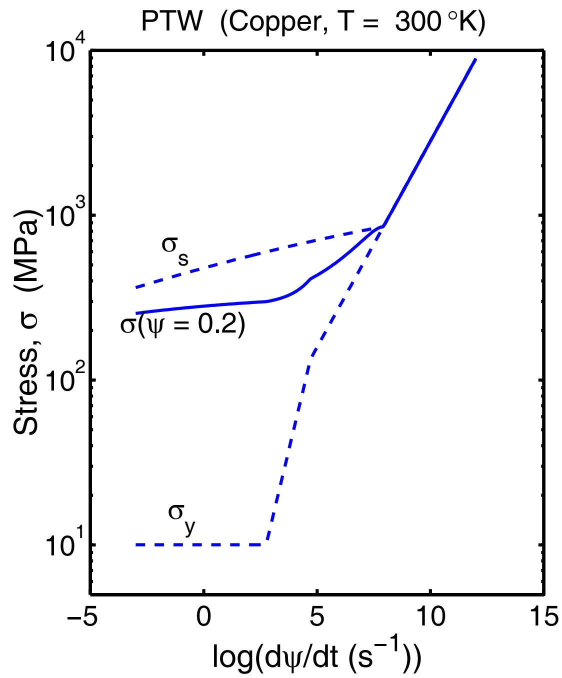

Figure 4. The von Mises stress plotted as a function of strain rate for copper at $T = 300\,{}^{\circ}K$ and $\psi = 0.2$. The lower and upper limits are shown as dashed lines.

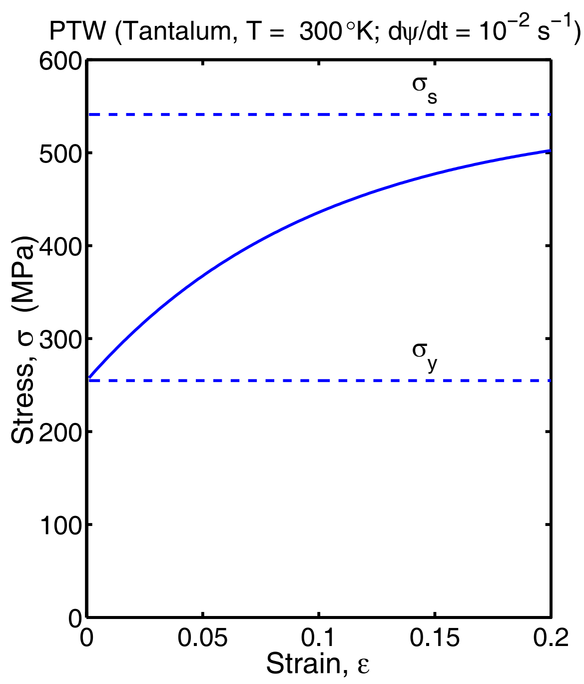

Figure 5. The von Mises stress plotted as a function of strain (denoted in the text by ψ, but often by ϵ) for tantalum at $T = 300\,{}^{\circ}K$ and $\dot{\psi} = 10^{-2}\,\mathrm{s}^{-1}$. The lower and upper limits for these conditions, given by Eqs. (11) and (9), are shown as dashed lines.

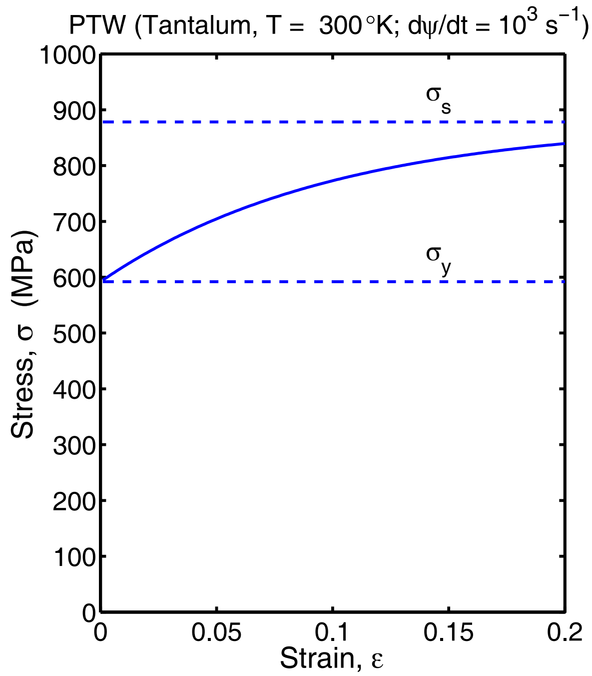

Figure 6. The von Mises stress plotted as a function of strain for tantalum at $T = 300\,^{\circ}K$ and $\dot{\psi} = 10^3\,\mathrm{s}^{-1}$. The lower and upper limits are shown as dashed lines.

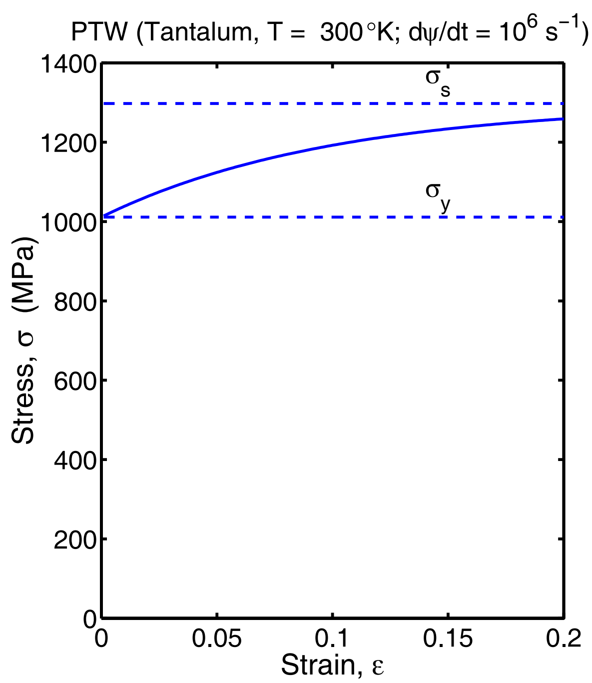

Figure 7. The von Mises stress plotted as a function of strain for tantalum at $T = 300\,^{\circ}K$ and $\dot{\psi} = 10^6\,\mathrm{s}^{-1}$. The lower and upper limits are shown as dashed lines.

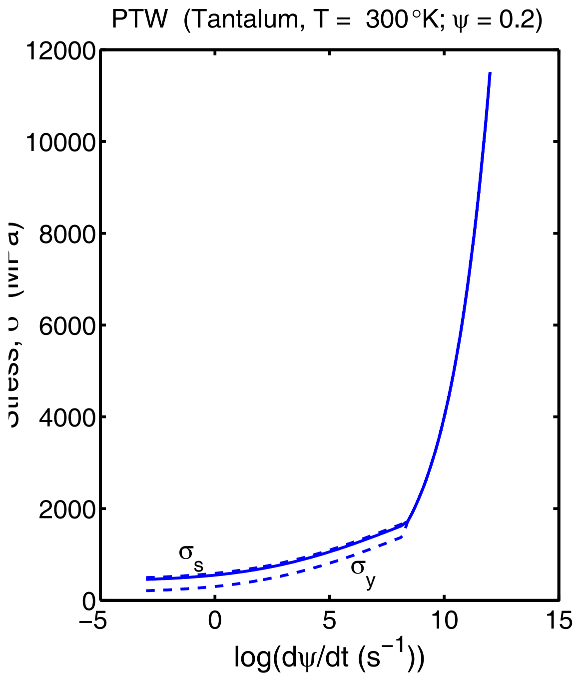

Figure 8. The von Mises stress plotted as a function of strain rate for tantalum at $T = 300\,°K$ and $\psi = 0.2$. The lower and upper limits, given by Eqs. (11) and (9), are shown as dashed lines.

REFERENCES

1. D. L. Preston, D. L. Tonks, and D. C. Wallace, "Model of plastic deformation for extreme loading conditions," *J. Appl. Phys.* **93**, pp. 211–220, 2003.
2. D. C. Wallace, "Irreversible thermodynamics of overdriven shocks in solids," *Phys. Rev. B* **24**, pp. 5597–5606, 1981.