Bayes Days 2000 at LANL

Three-Day Minicourse on Bayesian Analysis in Physics

Lectures presented by

Prof. Volker Dose
Max Planck Institute for Plasma Physics

Sponsored by
Enhanced Surveillance Program, Los Alamos National Laboratory

For more information, look on the web:
http://public.lanl.gov/kmh/course/BD2000.html

Organized by Ken Hanson, DX-3, 505-667-1402, kmh@lanl.gov

Energy confinement in Fusion devices

V. Dose, R. Preuss, W. von der Linden

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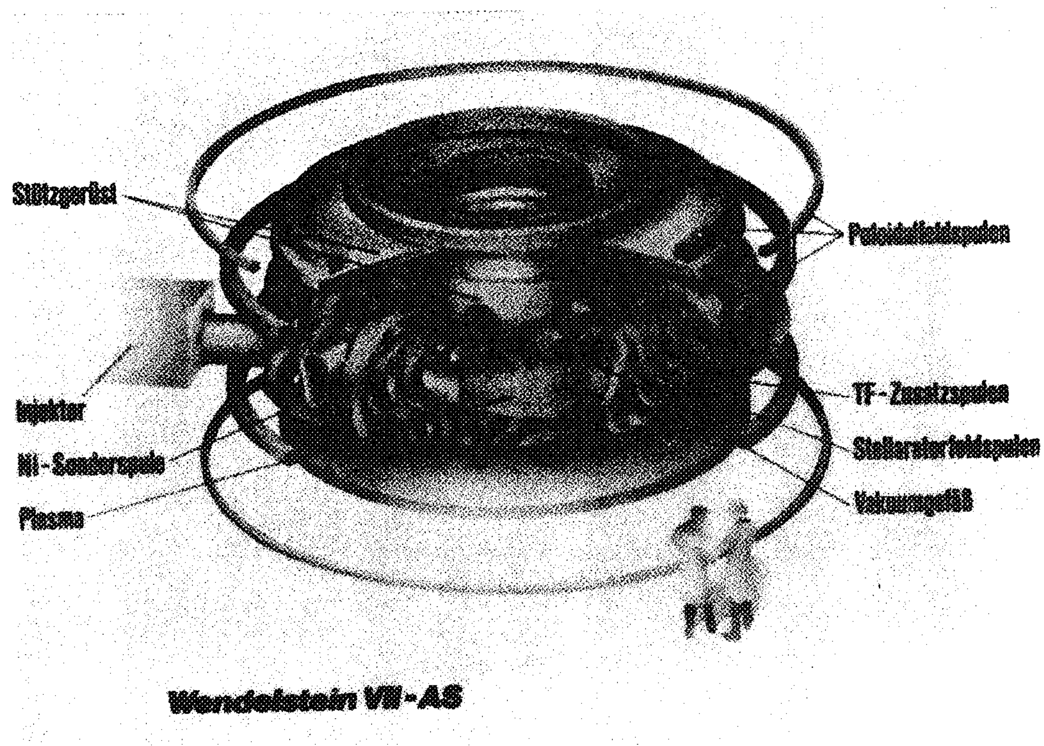
Large Fusion experiments



Plasma determined by:

$$\left.\begin{array}{ll} n & \text{particle density} \\ B & \text{magnetic field} \\ P & \text{heating power} \\ a & \text{minor radius} \\ R & \text{major radius} \end{array}\right\} \rightarrow \begin{array}{c} \text{energy} \\ \text{confinement} \\ W^{exp} \end{array}$$

Scaling law: functional form unknown

$$W_i^{theo} = e^{\alpha_c} n_i^{\alpha_n} B_i^{\alpha_B} P_i^{\alpha_P} a_i^{\alpha_a} + \varepsilon_i$$

Plasma equations

$$\frac{\partial f}{\partial t} + \vec{v}\ \mathrm{grad}_x f + \vec{a}\ \mathrm{grad}_v f = \left(\frac{\partial f}{\partial t}\right)_{collisions}$$

$$\vec{a} = \frac{e}{m}\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\mathrm{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \mathrm{rot}\vec{B} = \mu_0\ \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\vec{j} = e\int \vec{v} f(\vec{x},\vec{v})\mathrm{d}^3 v$$

$$E = \int \frac{mv^2}{2} f(\vec{x},\vec{v})\mathrm{d}^3 v = \frac{W}{\pi a^2 \cdot 2\pi R}$$

$$Q^{\nearrow} = \int \vec{v}\ \frac{m}{2} v^2 f(\vec{x},\vec{v})\mathrm{d}^3 v = \frac{P^{\swarrow}}{2\pi a \cdot 2\pi R}$$

$$n = \int f(\vec{x},\vec{v})\mathrm{d}^3 v$$

Transform

$$f \to \alpha f\ , \qquad \vec{x} \to \gamma \vec{x}\ , \qquad \vec{v} \to \beta \vec{v}$$

Similarity transformations

$$f \to \alpha f \ , \qquad \vec{x} \to \gamma \vec{x} \ , \qquad \vec{v} \to \beta \vec{v}$$

$$P \to \alpha \beta^6 \gamma \tilde{P} \ , \qquad n \to \alpha \beta^3 \tilde{n} \ ,$$

$$W \to \alpha \beta^5 \gamma^2 \tilde{W} \ , \qquad B \to \frac{\beta}{\gamma} \tilde{B} \ , \qquad a \to \gamma \tilde{a}$$

$$W = c_{pqrs} n^p B^r P^q a^s$$

$$\begin{aligned} 1 &= p + q \\ 5 &= 3p + r + 6q \\ 2 &= -r + q + s \end{aligned}$$

$$\begin{aligned} W &= c_q \ n^{1-q} \ P^q \ B^{2-3q} \ a^{4-4q} \\ &\sim nB^2 a^4 \left\{ \frac{P}{nB^3 a^4} \right\}^q \end{aligned}$$

CT Model	x_1	x_2	x_3	DOF
1 collisionless low-β	x	0	0	1
3 collisionless high-β	x	0	z	2
4 collisional high-β	x	y	z	3

$$W^{theo} \propto n_i a_i^4 R B_i^2 \left(\frac{P_i}{n_i a_i^4 R B_i^3}\right)^{x_1} \left(\frac{a_i^3 B_i^4}{n_i}\right)^{x_2} \left(\frac{1}{n_i a_i^2}\right)^{x_3}$$

$$= c f\left(n_i, B_i, P_i, a_i; \vec{x}\right)$$

$$\boxed{\vec{W}^{theo} = \sum_{k=1}^{E=3} c_k \vec{f}\left(\vec{x}_k\right)}$$

$$\begin{aligned} W_i^{theo}(\qquad) = \; & c_1 \; n_i a_i^4 B_i^2 \left(\frac{P_i}{n_i a_i^4 B_i^3}\right)^{x_1} \left(\frac{a_i^3 B_i^4}{n_i}\right)^{y_1} \\ + \; & c_2 \; n_i a_i^4 B_i^2 \left(\frac{P_i}{n_i a_i^4 B_i^3}\right)^{x_2} \left(\frac{a_i^3 B_i^4}{n_i}\right)^{y_2} \\ + \; & c_3 \; n_i a_i^4 B_i^2 \left(\frac{P_i}{n_i a_i^4 B_i^3}\right)^{x_3} \left(\frac{a_i^3 B_i^4}{n_i}\right)^{y_3} \end{aligned}$$

Likelihood function

$$p(\vec{W}^{exp}|\omega, \vec{c}, \vec{x}, E, M_j, \vec{\sigma}, I) = \left(\frac{\omega}{2\pi}\right)^{\frac{N}{2}} \prod_i^N \sigma_i^{-1}$$

$$\cdot \exp\left\{-\omega \sum_{i=1}^{N} \frac{\left(W_i^{exp} - \qquad\qquad\right)^2}{2\sigma_i^2}\right\}$$

Model comparison

$$\frac{p(M_j|\vec{W}^{exp}, \vec{\sigma}, I)}{p(M_k|\vec{W}^{exp}, \vec{\sigma}, I)} = \underbrace{\frac{p(M_j|\vec{\sigma}, I)}{p(M_k|\vec{\sigma}, I)}}_{=1} \frac{p(\vec{W}^{exp}|M_j, \vec{\sigma}, I)}{p(\vec{W}^{exp}|M_k, \vec{\sigma}, I)}$$

$$p(\vec{W}^{exp}|M_j, \vec{\sigma}, I) = \sum_E p(E|M_j, I)p(\vec{W}^{exp}|E, M_j, \vec{\sigma}, I)$$

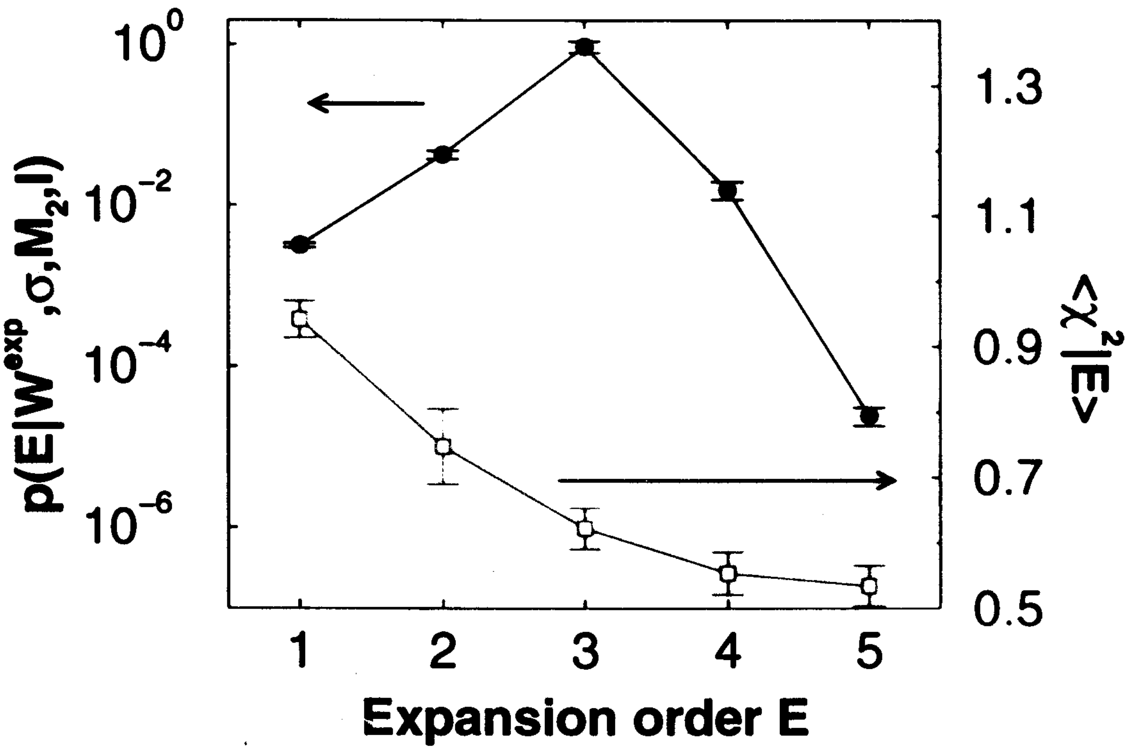
$$p(\vec{W}^{exp}|E, M_j, \vec{\sigma}, I) = \int p(\vec{W}^{exp}|\omega, \vec{c}, \vec{x}, E, M_j, \vec{\sigma}, I)$$
$$\times p(\omega, \vec{x}, \vec{c}|E, M_j, I)\ \mu(\omega, \vec{x}, \vec{c})\ d\omega\ d\vec{c}\ d\vec{x}$$

Model comparison

CT Model	$p(M_j \mid \boldsymbol{W}^{exp}, \boldsymbol{\sigma}, I)$
1. collisionless low-β	$4 \times 10^{-12}\%$
	99.7%
3. collisionless high-β	0.25%
4. collisional high-β	0.025%

Expansion order

$$p(E \mid \vec{W}^{exp}, M_j, \vec{\sigma}, I) = \frac{p(E \mid M_j, I)\, p(\vec{W}^{exp} \mid E, M_j, \vec{\sigma}, I)}{p(\vec{W}^{exp} \mid M_j, \vec{\sigma}, I)}$$

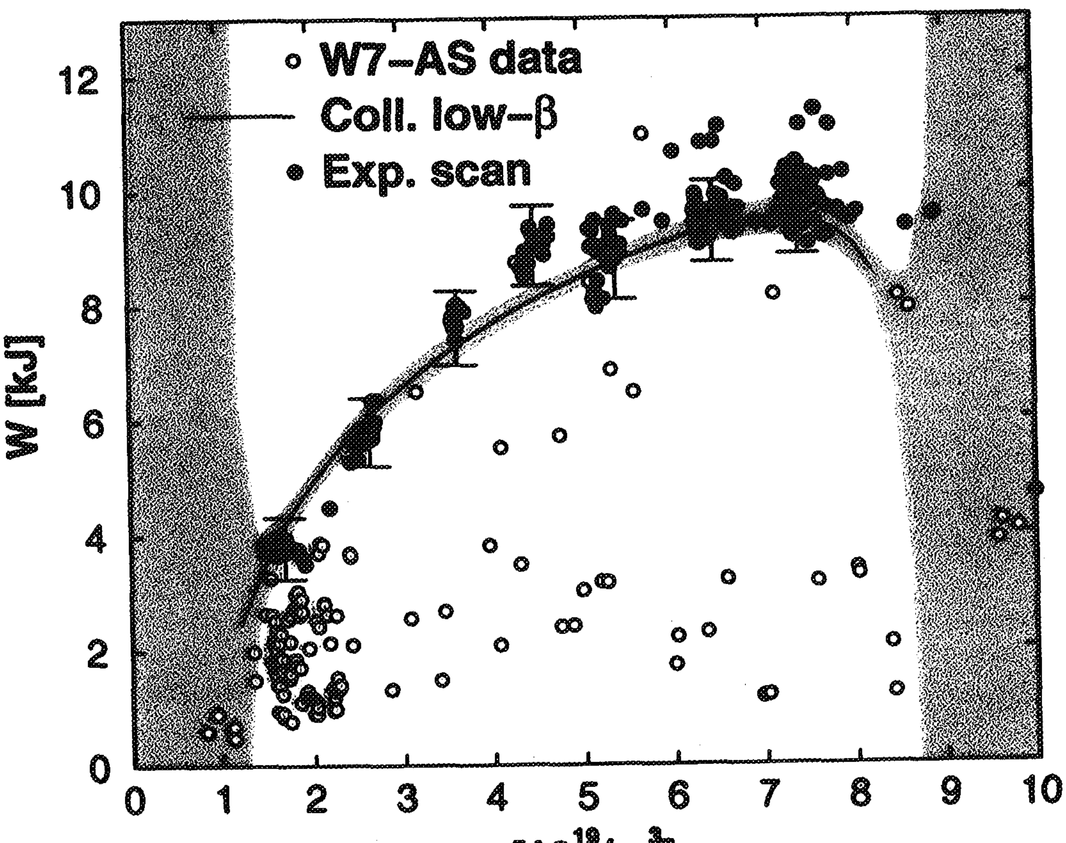


Density scan

Energy content as function of density

$$\langle W \rangle = \frac{\int dW \; W \; p(W|\vec{W}^{exp}, \vec{v}, M_j, \vec{\sigma}, I)}{\int dW \; p(W|\vec{W}^{exp}, \vec{v}, M_j, \vec{\sigma}, I)}$$

with $\vec{v} = (\hat{n}, \hat{B}, \hat{P}, \hat{a})$



$(\hat{B} = 2.5\text{T}, \; \hat{P} = 0.45\text{MW}, \; \hat{a} = 0.176\text{m})$

Power scan

Energy content as function of absorbed power
($\hat{n} = 2.4 \cdot 10^{19}\mathrm{m}^{-3}$, $\hat{B} = 2.5\mathrm{T}$, $\hat{a} = 0.176\mathrm{m}$)